A MONTHLY PUBLICATION BY COMPUTER SCIENCES CORPORATION / VOL. 1, NO. 3

CSC REPORT

TIME-ORIENTED FORTRAN

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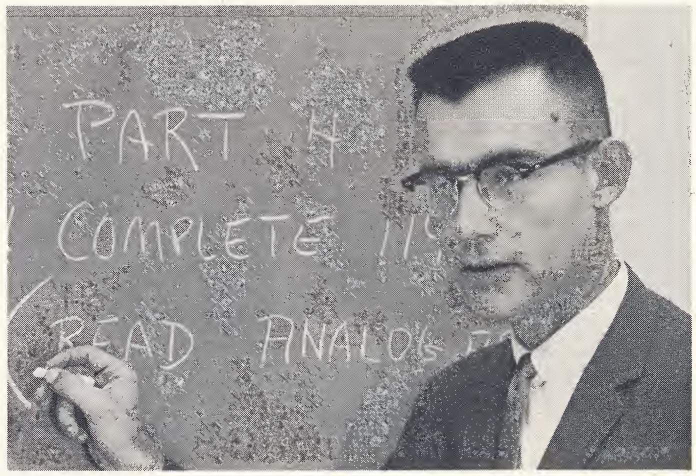


Rod McMillan of CSC's Houston Division has a broad range of over nine years real time computer experience. He was formerly manager of Programing Support for General Electric Process Computers and has also been employed by major users of edp equipment.

By ROD Mc MILLAN, CSC Houston Division

When the Houston Division of CSC was organized last year, one of its planned areas of emphasis was military and industrial real time data acquisition and control. An initial investigation of available software indicated the lack of a compiler which could handle interrupt logic, parallel programing, or provide time oriented statements within the framework of its own language. Several existing compilers could communicate with a monitor system but that monitor was fixed in concept and the total effect was a system which lacked the flexibility needed for a wide acceptance in the real time field.

To fill this need, the Houston staff developed Time-Oriented FORTRAN. This was chiefly a language development. It was believed that the language should be a natural extension of FORTRAN, although the same concepts could be applied to other languages with only a change in the form of the statements. Basically, Time-Oriented FORTRAN allows the programer to operate peripherals simultaneously, recognize automatic interrupts, divide his program into its logical parts, control the allocation of memory, and make direct reference to time or time interval variables. This extension of the language usually permits the programer to write his entire program in the compiler language.

The extension of FORTRAN can be divided into four sections:

Interrupt statements
Transfer statements to the point of interrupt
Parallel input/output statements
Memory organization

It should be noted that all present FORTRAN statements are retained and only the additions will be presented.

In order to take action upon hardware interrupts, the compiler generates a routine, CONTROL, which does the housekeeping associated with the following statements:

WHEN	$(I_1, \ldots I_n)$	$(N_1, \ldots N_n)$
PERMIT	$(I_1, \ldots I_n)$	
DELAY	$(I_1, \ldots I_n)$	
TALLY	$(I_1, \ldots I_n)$	
END TALLY	$(I_1, \ldots I_n)$	$(A_1, \ldots A_n)$

In the above (I_j) are fixed point constants, (N_j) are statement labels, and (A_j) are fixed point variables. The working of these statements is as follows: WHEN tells CONTROL that as interrupt I_j occurs, the program flow should transfer to the statement whose label is N_j. PERMIT enables the listed interrupts to occur and DELAY prevents the interrupts both according to the hardware restrictions of the computer. Since many computers group the interrupts for PERMIT or DELAY, the programer may wish to avoid hardware restrictions and use a TALLY statement to handle them individually. TALLY allows the interrupt I_j to occur physically. However, instead of the logical transfer of control to N_j, the corresponding variable A_j is incremented by one. An END TALLY stops this procedure and sets the

status to PERMIT.

TOF provides for the interruption of the program and its subsequent return without restricting the order of return. In order to achieve this objective, the program must be conceived as a collection of PARTS. These PARTS may be subroutines, functions, subprograms, etc., as the programer chooses. For this purpose, the following statements are available:

SET I
ENTRY TO I
RESUME I
+N
—N

where I is a fixed point constant which identifies the PART, and N is the object label in an immediate transfer statement.

Execution of the SET I designates PART I as the part being executed until another SET statement is encountered. CONTROL then knows which part is being executed and associates any point of interrupt with the current PART. RESUME I tells CONTROL to continue PART I at its associated point of interrupt. +N, appearing in a transfer control statement, simulates an automatic interrupt and has the effect of a mark-place-and-transfer instruction. —N, used in a manner similar to +N, initializes the current PART so that any subsequent RESUME will go to the statement which followed ENTRY TO I during compilation.

There are three statements available for controlling the transmission of information during execution of the object program. These are:

INITIATE I READ (UNIT, F) LIST
PRIORITY K, N
QUIT I

In addition, there are two transfer control statements associated with the Input/Output:

IF COMPLETE (I) N_1, N_2, N_3
WHEN COMPLETE (I) N

The INITIATE takes the place of the normal FORTRAN I/O statement, referred hereafter as an I/O COMMAND. It should be noted that there is really little change in the basic form. The FORTRAN I/O COMMAND is preceded by an identifier I (integer variable) as the object of an INITIATE statement. The PRIORITY statement assigns a priority K to the I/O statement whose LABEL is N. The QUIT statement aborts the I/O command whose identifier is I. The IF COMPLETE tests the status of I/O command I, and transfers to N_1 if the command is complete, N_2 if not complete and N_3 if malfunction prevents completion of the command. WHEN COMPLETE is a future GO TO which simulates an external interrupt when I/O command I is complete.

The actual execution of the I/O commands is by an I/O MONITOR which is generated by the compiler according to the demands of the program.

Further statements which are compiler directives governing the allocation of memory are largely machine dependent. Time-Oriented FORTRAN was developed for industrial and military real time applications. These programs often exceed in size the available core store. This requires that many parts of the program utilize the same memory for execution but be permanently stored in unique mass memory areas. In current compilers the practice of relocation is acceptable for off-line processing through an executive system. Real time applications, however, do not allow for the luxury of relocation when programs are called into core or are dynamically relocated using an indexing scheme. The compiler, therefore, must have information relating to the programer's plan of memory or must allow for a post compile assembly. The general form of this information is the statement MEMORY BLOCK JA, KA, L where JA is a core address, absolute or symbolic, KA is a drum address, absolute or symbolic, and L is an unsigned fixed point constant which tells the compiler the estimated size of the block.

Implementation of this language may vary according to the object computer but the general philosophy and approach to a time oriented language are applicable to the general class of on line, real time computers. The important consideration is that the language is capable of handling the entire program and is a sufficiently powerful tool for the applications oriented, real time programer.

CSC PLANS PUBLIC OFFERING

CSC has filed a registration statement with the Securities and Exchange Commission relating to registration of 200,000 shares of Common Stock planned for a public offering in September.

Manager of the offering is White, Weld and Co. Incorporated.

LOCKHEED AWARDS PROGRAMING CONTRACT

A major contract for computer programing support has been received by CSC from Lockheed Missiles and Space Company, Sunnyvale, Calif.

Computer programing will be accomplished for such large scale equipment as the IBM 7090/7094 computers. Programs will be part of an Automatic Data Accumulation system for Lockheed, Sunnyvale facilities.

Work will be performed by personnel at CSC's new Palo Alto office.

LAYSER APPOINTED VICE PRESIDENT

David K. Layser has been appointed Vice President, Finance. Joining CSC in February, 1962, Mr. Layser has served as treasurer and controller as well as a member of the Board of Directors. He was formerly treasurer and controller of the Hiller Aircraft Corporation and held a similar position with Weston Hydraulics, Ltd.

Mr. Layser's background of more than ten years professional experience in financial management also includes such positions as senior accountant with the national public accounting firms of Touche, Ross, Bailey and Smart, and Peat Marwick Mitchell and Co.

A member of the California Society of Public Accountants and National Association of Accountants, Mr. Layser received his Master of Business Administration degree in 1952 at the University of California.

CSC OPENS SAN FRANCISCO OFFICE

A new office was opened by CSC this month in the San Francisco Bay Area. Located at 2500 El Camino Real in Palo Alto, the new facility will be managed by Ed Dodge.

As the fifth of CSC's offices, services will be provided to commercial, military and scientific users in northern California covering the entire range of CSC's capabilities.

Staff members presently assigned to the San Francisco office include Joe Donegan, Robert Gebelein, Ken Harnquist, Eugene Johnson, and Steve Showler. The staff is presently being increased by transfer of other personnel and local recruiting.

FOREIGN DIGNITARIES VISIT LA & HOUSTON

Computing dignitaries from The Netherlands, Switzerland, Norway and Argentina were hosted last month in the Los Angeles and Houston offices of CSC.

In Los Angeles, representatives from the Phillips Corporation, Eindhoven, The Netherlands, held discussions with CSC vice president Roy Nutt. Included in this party were Dr. A. J. W. Duijzestijn, Head of Phillips Laboratory; P. Bejager, Chief of EDP Center, and A. Neeuwis, Chief of Administrative Organization.

Hosting a group from Switzerland was Service Bureau manager Dan Mason. This party included Dr. Hans K. Oppliger-Burger, Head of Coordinating Office for Automation, and Alfred Schai, Swiss Federal Institute of Technology, Zurich.

From the Norwegian Computing Center in Oslo, Bard Haerland and Bernard Hausner have been engaged in discussions with staff assistant Ray Bebo.

At Houston, Horacio C. Reggini of Fernandez Long y Reggini, Ingenieros Consultores, Buenos Aires, visited recently with division manager Hal Leone.

STUDENTS TOUR CSC FACILITIES

A program of inviting students from Los Angeles schools to tour CSC facilities has been initiated by EAM supervisor Bill Fahy with the assistance of Ray Bebo and Dick Lansdale.

During the summer months, three groups have received briefings on computing fundamentals at CSC. Classes from South High in Torrance, Long Beach City College and Culver City High School have visited CSC. An extended program of student orientation has been planned for the fall.

INPUT

Richard Wise and Erwin Haltinner joined CSC's technical staff last month, bringing an additional seven man years experience to the company.

Assigned to the Systems Programing department, Mr. Wise was formerly employed by Systems Development Corporation since 1959 where he programed such machines as the IBM 7090, CDC 160A, 1604 and AN/FSQ-7 and 32. As a senior programer analyst, he has worked with the CLIP and JOVIAL languages.

Mr. Haltinner has been assigned to the Scientific Applications department. He was formerly with the Hercules Powder Company since 1960 and has served as group leader for programing of the Minuteman flight data reduction system.

CSC PROFILE

Shavian-witted Fred Braddock views life and its minor complexities as a sport of sorts.

As an alien to the smog-smitten shores of southern California and its national environs, Fred offers unusual contrast to the U. S. version of a computer programer. Born 25 years past in a city sometimes called Watford, near London, Fred has varied his professional growth from porridge boiler scraper to mathematician imbedded in atomic reactor projects. The intervening years have provided a variety of sport, study and occupational hazards the likes of which would frighten Lloyds of London.

While attending Cambridge University, Fred divided his formative years between track (4 minute, 52 second mile and winner of two cross-country meets); academic niceties (elected vice president of the Cambridge Mathematics Society), and a selection of such part time jobs as physicist, janitor, postman, stagehand, actor, liquor store clerk, barkeep, and laundry attendant. Despite or because of his extra curricular necessities, Fred graduated from Cambridge *with honours* in mathematics.

Following his formal academic training, Fred was employed by the Atomic Energy Division of General Electric Company in London (no affiliation with the U.S. version of GE) where he was initiated in the programing field working with several Ferranti machines. His specific area of application was investigation of control rod configurations for nuclear reactor cores. After several years in this position, Fred was recruited via Polar route for CSC's Scientific Applications department and has been involved in the writing of routines for the FORTRAN library.

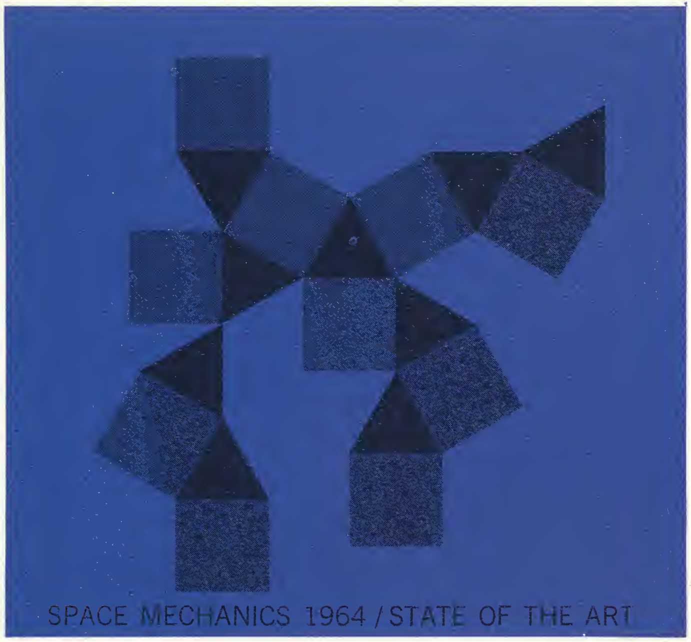
Retaining the flavor of his earlier years, Fred Braddock's hobbies consist of porcupine quill collecting, fox hunting, short story writing (this particular effort has netted him 18 shillings ($3) additional income), motor car rallying, steeplechasing, and sailing (in one competition, he turned over before reaching the starting line).

Perhaps it is because of his hobbies that as yet, Fred has been unable to find time for marriage and family. He requests applications from opposite numbers in tightly sealed, unmarked envelopes.

CSC REPORT — SEPTEMBER, 1963
Published by Computer Sciences Corporation
650 North Sepulveda Boulevard
El Segundo, California
Los Angeles - Houston - New York - London

A MONTHLY PUBLICATION BY COMPUTER SCIENCES CORPORATION / VOL. 2 NO. 2

CSC REPORT



Dr. Baker is one of the nation's foremost scientists in astrodynamics and aerodynamics. With a background of more than 13 years in the space sciences, Dr. Baker was formerly with the Lockheed Astrodynamic Research Center and has also served with the United States Air Force, Space Systems Division, and Douglas Aircraft. He is presently editor of the JOURNAL OF ASTRONAUTICAL SCIENCES.

By Dr. Robert M. L. Baker, Jr., Mathematical Analysis Department

From the enormous number of recent contributions to space mechanics, one can detect a gradually developing, overall tendency away from aloof theoretical orbit mechanics, which perhaps was emphasized in the late 50's by more practical research. We therefore find that the pacing problems in space mechanics which must be solved first are largely associated with specific astronautical systems rather than with the frontiers of theoretical research such as the three-body problem, special perturbations, etc. There has been a tremendous concentration of effort on specific lunar trajectories and the space mechanic research work in this area is down to fine circles. Not so with interplanetary trajectories, and in this field, broad parametric studies are still in vogue—perhaps because our nation's interplanetary space goal is not yet well defined.

Some of the major problem areas that are receiving attention are in the higher precision planetary ephemerides, astrodynamic constants, non-gravitational forces, orbit determination, and new computational techniques. The list can be extended, of course, but the foregoing serve as examples. We shall discuss each of these in turn and mention some of the efforts that have been mounted for their solution.

For a number of years, it has become obvious that the accuracy with which we could place a spacecraft on a given planet (especially Mars), if we were provided with its precise location, is greater than the accuracy with which we can define the position of the planet. Conventional astronomical ephemerides simply had never been faced with such a stringent accuracy requirement and, in consequence, were now in need of revision. This problem was first detected in the accomplishment of various projects by Jet Propulsion Laboratory and P. R. Peabody, Neal Block and others who have developed new, more accurate ephemeris calculations, correct within a few kilometers, that will meet present and future demands.

The area of astrodynamic constants has always been a most vital one and few space scientists now need to be convinced of the importance of such constants. There has been a major attack mounted on the determination of the yardstick of the solar system termed the "solar parallax." Soon, when Mars and Venus again assume favorable locations relative to the Earth, a multitude of new and better radar ranges to the planets (which yield values of the solar parallax) will be accomplished by JPL, MIT, Jodrell Bank, etc. It is especially hoped that a reconciliation will be made between the older optical-astronomical values and the newer electronic-astronautical values of this important quantity. The electronic procedure for defining the solar parallax is to measure the time required for electromagnetic radiation to traverse a distance already known in terms of the "astronomical unit." Roughly speaking, the astronomical unit is the mean distance from the Earth to the Sun. Since the distance to Venus, at any given time, is already well known in terms of astronomical units, one can determine the ratio (i.e., by multiplying the total transit time of the electromagnetic waves, to Venus and return, by the speed of light). Since the radar measurement is in terrestrial units such as kilometers, a direct ratio of the kilometer to the astronomical unit is obtained. The solar parallax $\pi_\odot$, is in turn defined as the inverse sine of the ratio of the Earth's equatorial radius (which is 6,378.165 kilometers) to the astronomical unit (now determined by radar to be 149,600,000 km). Thus we obtain $\pi_\odot$

$$\pi_\odot = \sin^{-1}\left(\frac{149{,}600{,}000}{6{,}378.165}\right) = 8.''794\ 1,$$

a value which is three orders of magnitude better than earlier optical-astronomical determinations of this same quantity by triangulation.

Other constants relate to the planets, the Earth, and the Earth-Moon System. G. F. MacDonald of UCLA has studied the geophysics of the planets, while G. F. Schilling of RAND Corporation has produced an excellent survey of the atmosphere of Mars. In the area of geocentric constants, the work at the Air Force Cambridge Research Laboratories is especially noteworthy and we cite the studies of O. W. Williams, Mahlon Hunt, and

Armando Mancini. Of course, these studies must be based upon the analyses of actual satellite data carried out by Izsak, Kaula, Kozai, and others. Turning to lunar constants we note the work of Makemson as well as that of Z. Kopal. Clearly, as the day approaches when we place a satellite on orbit about the Moon and then a man on the lunar surface itself, these "selenocentric" constants will enjoy a marked improvement. Basically, these constants fall into two categories, first, the geometrical ones such as a planet or moon's shape and size, and gravitational ones, which describe planet or moon's gravitational field. These latter constants can be arrived at either approximately by assuming a given internal structure and thereby establishing the external gravitational field or more exactly by observing the motion of an object passing through this field. Certain geometrical constants of recent vintage that are associated with Mars, Earth, Moon, and Venus are given in the following table.

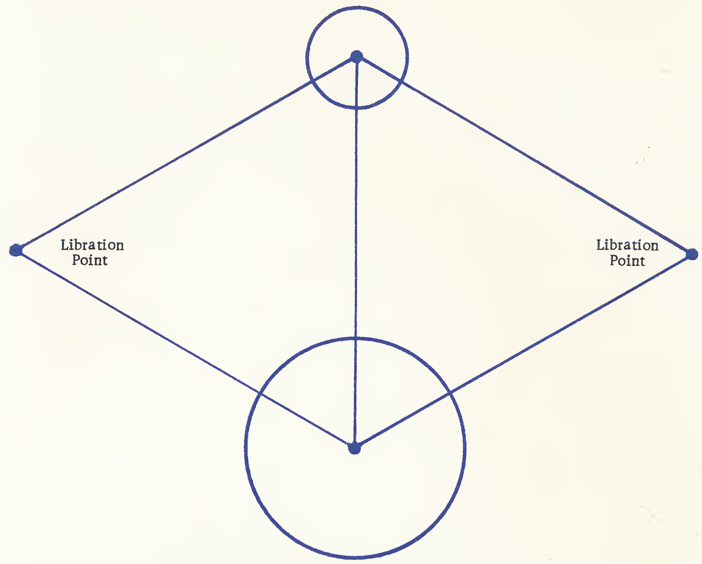
TABLE I
Geometrical Constants

	Equatorial Radius *(km)*	Flattening *(Ratio of the difference between the equatorial and Polar radius to the Equatorial Radius)*
MARS	3,415.0	1/192
EARTH	6,378.165	1/298.3
MOON	1,738.3	0.000,337,4
VENUS	6,100.0	0

Non-gravitational forces play an especially strong role in communication satellites, which often exhibit a large area to mass quotient and are therefore subject to radiation pressures and (if they are low enough) to atmospheric drag. The (as yet unpublished) works of I. Sugai at ITT and of Gordon Negus at Rome Air Development Command have demonstrated both the importance of these non-gravitational forces as well as the difficulties encountered in dealing with them. To cite but one of these difficulties, consider the influence of one portion of a spacecraft shading another on the calculation of radiation pressure—the problem is confounding. In the area of satellite drag, it is hoped that the research of Bernard Cohlan at UCLA will clarify one of the most confused and uncertain areas of astrodynamics and space mechanics. The literature here is indefinite and often contradictory.

With the never ending addition of new and more precise observational devices to our space ranges and with the ever increasing numbers of new spacecraft injected onto orbit, the demands for orbit determination are more and more rigorous. Faced directly with these problems is Joseph Siry at Goddard Space Flight Center, and his associates, Frank Lerch and Soar. At Goddard the concentration is on geocentric orbit determination, while at the Manned Spacecraft Center James F. Dalby, grapples with lunar trajectories, and at MSFC, Fletcher Kurtz, with William Miner and others must deal with lunar and interplanetary orbit determination.

As mentioned earlier, longer strides have been made in the case of lunar trajectory and lunar satellite orbit determination of late, due no doubt, to our national goal in the lunar area. As the term is used here, orbit determination is meant to be the definition, by means of observational data, of initial conditions or other constants, which can be utilized as a basis for predicting the future course of a spacecraft. The orbit determination process need be



no more accurate than the data upon which it is based. Thus, as the data become more accurate and ample, the orbit determination schemes must be updated to take maximum advantage of the improved data. Although space is truly vast and spacecraft are not likely to collide accidentally, the spacecraft orbiting near to the Earth do tend to clutter our space tracking systems with a tremendous amount of data. Therefore, our orbit determination schemes must be able to differentiate among data collected from a number of satellites and process them individually. This problem of discrimination becomes more difficult as the number of spacecraft on orbit steadily increases and our orbit determination schemes must keep pace.

The author is familiar with a number of new computational techniques that emerged in 1963. Once again, the reader should recognize that the items mentioned are not meant to be comprehensive, but, simply indicative of progress. Herrick's universal variables, which allow for a standard treatment of orbits having any conic form, met with some difficulty when applied to numerical computations. The work of E. Pitkin, especially, has shown us the way (through nesting the series, etc.) to make use of, and take maximum advantage from these variables. In a rather unrelated area to Pitkin's work, we find linearized theory and solution of linearized equations by Laplace transforms (a rather old and standard procedure in control theory) applied to astrodynamical problems by K. Forster. Such an approach is somewhat novel in analytical orbit prediction (the general term for which is "general perturbations") and opens up new avenues not only in orbit prediction, but also in the design of spacecraft orbits.

General perturbation procedures can be exceedingly elaborate and detailed. An example of the requirement for such detail is André Deprits' work on the three-body problem at the Boeing Scientific Research Laboratories. In contrast to Forster's work, in which one of the goals is, e.g., the engineering design of a solar sailing satellite and where a linearizing approximation is perfectly valid and necessary, the work of Deprits must involve a very high order of precision. Deprits is one of many researchers looking at the so-called libration point of the three-body problem. This point is roughly at the vertex of an equilateral triangle, the

other vertices of which are occupied by two or more ponderous bodies, one of which is considerably more massive than the other. If just the correct velocity is given to an infinitesimal body occupying the other vertex (at the libration point) such a body will remain "near" this point in space for a very long period. The exact orbit of this third body and how "near" it remains to the exact orbit libration (triangle point) is the subject of Deprits', Eugene Rabe's, and W. H. Michael's work. Eventually a spacecraft may be placed at such a libration point near the Earth (in which case the Earth and the Moon will occupy the other vertices of the equilateral triangle). Perhaps, as some believe, Nature has already deposited some "debris" at such a point in space. In fact some observers have even reported a cloud of such cosmic dust at just this point in space.

In the preceding summary of research, the concentration has primarily been upon the research work accomplished by government laboratories. Of course, there are also a number of very active research groups in private industry. Most of these are oriented to given companies' space projects and their contributions are usually made in rather specialized areas.

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COMPUTAX PLANS SET FOR 1964 RETURNS

Plans for marketing and implementation of CSC's new service to accountants, COMPUTAX, have been confirmed for the preparation of individual 1964 federal and state tax returns.

For accountants in the states of California, New York, Connecticut, New Jersey, and Texas, CSC will provide rapid compilation of taxpayers' data and produce completed copies of tax returns and a diagnostic report pointing out problem areas such as deductions in excess of allowable amounts, underpayment of current year's estimated tax, potential tax savings from filing separate returns, inconsistencies in data, etc.

Last year, 43,000 individual tax returns were processed for residents in the state of California without a single mathematical error. The CSC program was reported in *Business Week, The Wall Street Journal, Journal of Accountancy* and numerous trade journals in the computing field.

CSC RECEIVES $230,000 DOD CONTRACT

CSC has been awarded a $230,000 programming contract from the Department of Defense.

The new contract provides for the design and development of a machine-independent ALGOL translator system which will permit computer users to prepare and execute programs for a variety of small, medium, and large scale machines including the IBM 7094, UNIVAC 490, CDC 3600, and SDS 910. The ALGOL translator will assist in the effective use of computer systems by expediting assignments of jobs to machines which may not be fully occupied at the time a job assignment is made.

The CSC system represents an important stride in advancing the techniques of automatic programming. The system incorporates the efficiency of hand-produced translator programs and the flexibility and power of "syntax driven" translators. In addition, the CSC system provides a guarantee of standardization of the defined language, since it incorporates an optimized tabulated description of ALGOL which, by itself, serves to provide a description of the language and its translator.

ON LINE, REMOTE DEMOS SET NEW RECORD

A record-breaking number of CSC Service Bureau demonstrations was held last month as more than 20 organizations toured CSC with special emphasis given to on line, remote communications. A few of the firms viewing demonstrations of 1004/1107 capabilities included Tidewater Oil, Fourth Army, Control Data Corp., Bendix, Pacific Division, American Cement Co., Mattel Toy Co., Hughes Aircraft, Lockheed, Burroughs, Teledyne Corp., Lear Siegler, Logicon, and the French Center of Operations Research, Paris.

SCIENTIFIC APPLICATIONS WINS NEW AWARDS

CSC's Scientific Applications Department last month announced new contracts and extensions from Lear Siegler, Sandia Corp., Telemetrics Inc., Electronic Specialty, Computech Associates, and IBM.

BROWN & ROOT AWARD NEW CPM CONTRACT

Brown & Root, Inc., Houston, has awarded CSC a new contract for updating and modification of CPM programming on a large scale computer. In addition, CSC is assisting in the operation of the company's service center. Brown & Root is one of the country's largest construction firms. They are presently serving as general manager for Project Mohole which involves drilling through the earth's crust in the Caribbean.

CSC REPORT — AUGUST 1964
Published by Computer Sciences Corporation
650 North Sepulveda Boulevard
El Segundo, California
Los Angeles/San Francisco/Houston/Washington/London

Proc. ACM 66

Digital generation of continuous, filtered Gaussian noise

by K. E. TIERNAN and R. G. DIETRICH
Raytheon Company
Bedford, Massachusetts

INTRODUCTION

Filtered white noise may be represented as a train of impulse responses of the noise filter. Use is made of Campbell's theorem to show that the probability density function of the noise amplitude is Gaussian when the number of impulse responses becomes large, and the impulse responses are equally likely positive or negative and are Poisson distributed along the time axis. By taking advantage of the property

$$e^{-\alpha(t-t_1)} = e^{-\alpha(t-t_i)} e^{-\alpha(t_i-t_1)} \qquad t_1 \leqslant t_i \leqslant t$$

it will be shown that the output of a simple RC filter can be represented as

$$O(t) = \left[O(t_n) \pm c \right] e^{-\alpha(t-t_n)} \qquad t_n < t \leqslant t_{n+1}$$

This result is easily extended to include any linear filter.

Description of noise generator

The digital noise generator described here is analogous to applying a sequence of unit impulses which are Poisson distributed along the time axis, to a low pass filter. Assume for illustrative purposes that the filter has an impulse response:

$$h(t) = ce^{-\alpha t} \qquad (1)$$

and the impulses are equally likely positive or negative with an average of n impulses occurring in a unit time interval. The spectral level of the resultant noise is controlled with the gain constant c, given by $c = \sqrt{K/n}$ where K is the desired double sided zero frequency spectral density in units^2/cps. A sketch of the system is given in Figure 1.

The waiting times, $t_i - t_{i-1}$, between successive Poisson distributed impulses* are independent random variables with an exponential probability density function. A very simple means of digitally generating such random numbers is presented in Figure 2[1]. The basic principle involved is similar to flipping a coin and computing the time to the nth head. The spacing in time between successive heads has an exponential probability density function. Approximately 80,000 samples were obtained using this technique and a histogram was plotted of the results in Figure 3. Also shown in this figure is a theoretical exponential probability density function.

The output of the filter after the nth impulse has occurred can be written as:

$$O(t) = \left[O(t_n) \pm c \right] e^{-\alpha(t-t_n)} \qquad t_n < t \leqslant t_{n+1} \qquad (2)$$

*No attempts were made to generate impulses as such. Rather, only the response of a low pass filter (or filters) to an applied impulse was generated. In the sequel, although reference may be made to the generation of impulses, only the impulse response of a filter is ever actually generated.

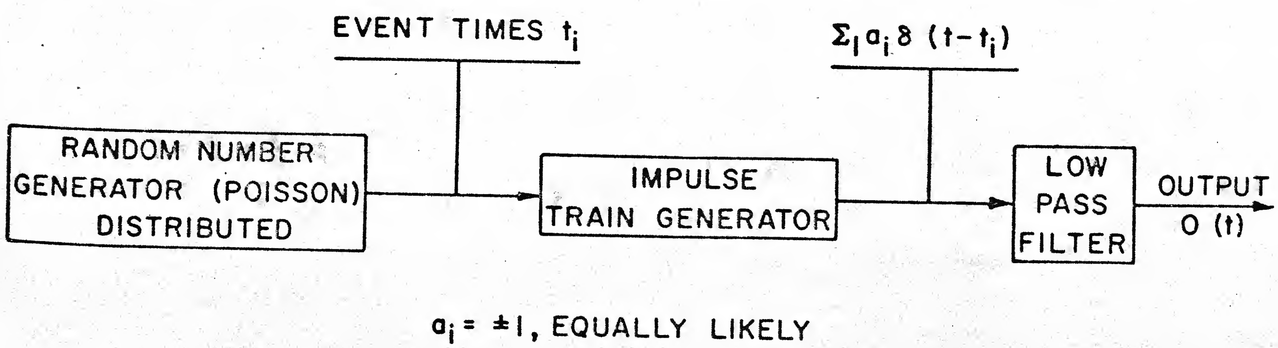

Figure 1 — An equivalent model of the digital noise generator

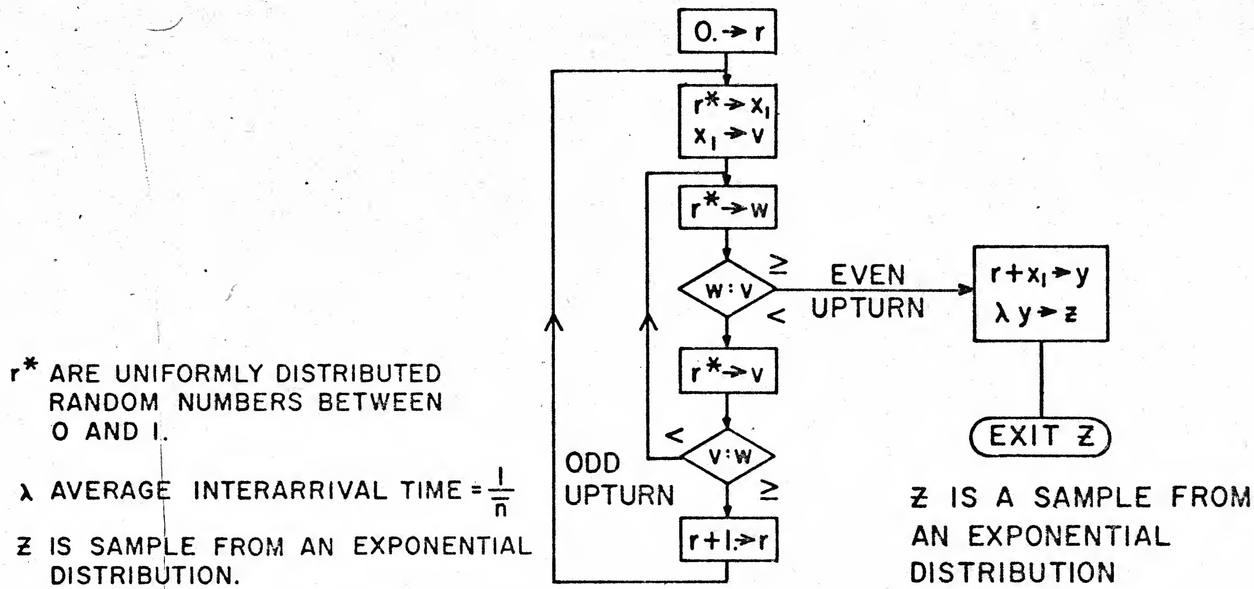

Figure 2 — Generation of exponentially distributed random numbers

where

t_n = time of occurrence of n^{th} impulse
$-\alpha$ = pole of the filter
$c = \sqrt{K/n}$ as previously defined

$$O(t_1) = \pm\, c e^{-\alpha(t-t_1)}$$

the $\pm$ signs are equally likely

The general case

Although the previous discussion has been presented assuming a first order time invariant filter, the approach is easily extended to any linear filter which can be represented as

$$O(t) = A(t)\, O(t) + B(t)\, i\,(t) \qquad (3)$$

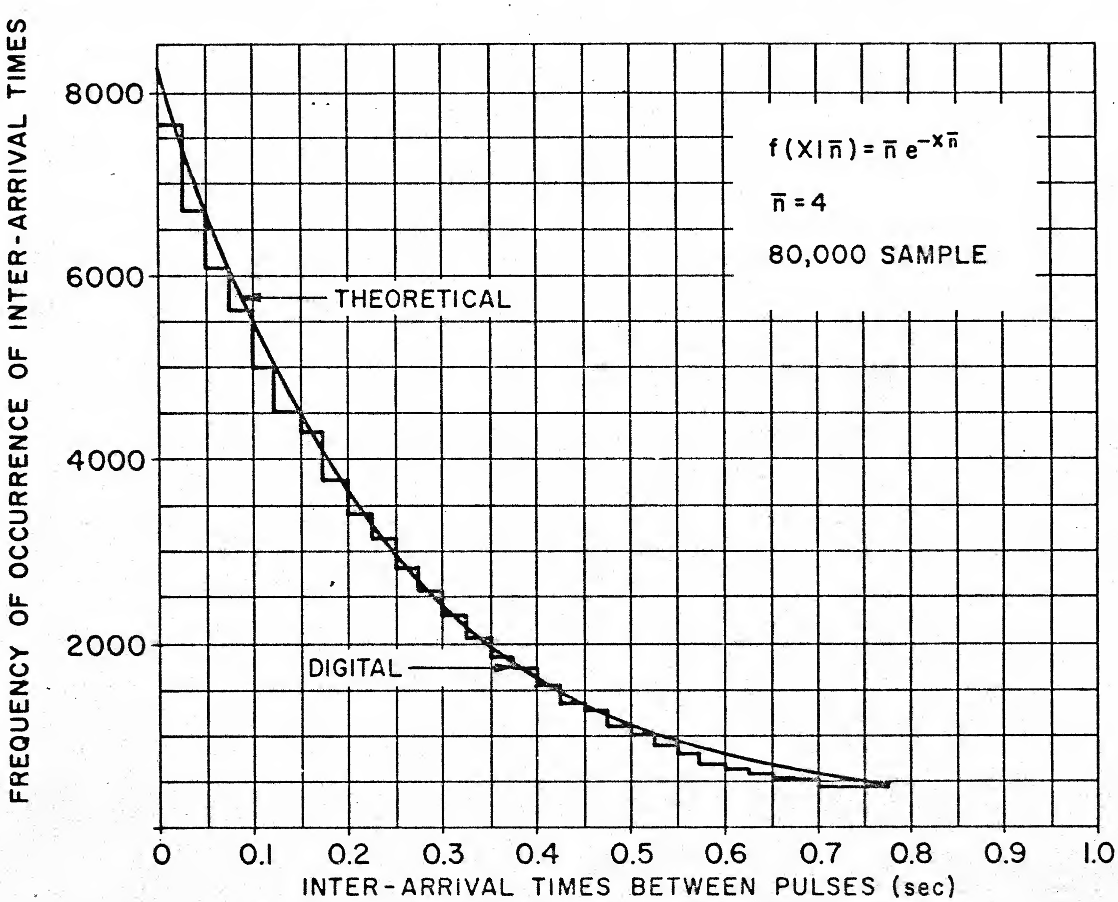

Figure 3 — Histogram of inter-arrival times between pulses with an exponential density function

where $A(t)$ and $B(t)$ are matrices and $O(t)$ and $i(t)$ are vectors; here $i(t)$ represents the input noise impulse train. In terms of the transition matrix, $\Psi(t, \tau)$ the solution for the output noise $O(t)$ is

$$O(t) = \Psi(t, t_1)\, O(t_1) + \int_{t_1}^{t} \Psi(t, t_1)\Psi^{-1}(t_1, \tau)B(\tau)i(\tau)d\tau \qquad (4)$$

In the case where the input $i(\tau)$ is a train of unit impulses such that the most recent impulse occurred at $t = t_n$, then

$$O(t) = \Psi(t, t_n)\left[O(t_n) \pm B(t_n)\right]^{**} \quad t_n < t \leqslant t_{n+1} \qquad (5)$$

where t_{n+1} is the time at which the next impulse occurs. If the matrix $A(t)$ is a matrix of constants, then the solution may be written as

$$O(t) = e^{A(t-t_n)}\left[O(t_n) \pm B(t_n)\right] \qquad (6)$$

Moreover, if the characteristic values of A are distinct, then

$$e^{A(t-t_n)} = T \begin{bmatrix} e^{\alpha_1(t-t_n)} & & & O \\ & e^{\alpha_2(t-t_n)} & & \\ & & \ddots & \\ O & & & e^{\alpha_k(t-t_n)} \end{bmatrix} T^T \qquad (7)$$

where

$$A = T \begin{bmatrix} \alpha_1 & & & O \\ & \alpha_2 & & \\ & & \ddots & \\ O & & & \alpha_k \end{bmatrix} T^T$$

and T is the orthogonal matrix that diagonalizes A.

For clarity, consider the single order filter shown in Figure 4 which may be represented by

$$\dot{O}(t) = -\alpha O(t) + i(t);$$

here $A = [-\alpha]$, $B(t) = [1]$. The output $O(t)$ can be written as (assuming the first impulse occurs at $t = O$):

$$O(t) = e^{-\alpha t} \qquad 0 < t \leqslant t_1 \qquad (8)$$

$$\left.\begin{aligned} O(t) &= e^{-\alpha t} \pm e^{-\alpha(t-t_1)} \\ &= \left[e^{-\alpha t_1} \pm 1\right] e^{-\alpha(t-t_1)} \end{aligned}\right\} t_1 < t \leqslant t_2 \qquad (9)$$

$$\left.\begin{aligned} O(t) &= (e^{-\alpha t_1} \pm 1)\, e^{-\alpha(t-t_1)} \pm e^{-\alpha(t-t_2)} \\ &= \left[(e^{-\alpha t_1} \pm 1)\, e^{-\alpha(t_2-t_1)} \pm 1\right] e^{-\alpha(t-t_2)} \\ &= (O(t_2) \pm 1)\, e^{-\alpha(t-t_2)} \end{aligned}\right\} t_2 < t \leqslant t_3 \qquad (10)$$

$$\cdots\cdots\cdots$$

$$O(t) = (O(t_n) \pm 1)\, e^{-\alpha(t-t_n)} \quad t_n < t \leqslant t_{n+1} \qquad (11)$$

Equation (9) indicates that one can compute the value of $O(t)$ at some time $t(t_1 \leqslant t < t_2)$ by computing the value of the exponential at the two distinct values αt and $\alpha(t - t_1)$ or by computing the value of the exponential for $(\alpha(t - t_1))$ and weighting this result by $(e^{-\alpha t_1} \pm 1)$. Since this coefficient represents the value of $O(t)$ at time $t = t_1$ plus the effect of a unit impulse occurring at $t = t_1$, then by continuing this procedure, it is easy to see that $O(t)$ can be calculated in the interval $(t_n \leqslant t < t_{n+1})$ by computing $e^{-\alpha(t-t_n)}$ and weighting by $O(t_n)$ plus the effect of a unit impulse occurring at $t = t_n$. The implication of this procedure is of paramount importance with regard to digital computations because $O(t)$ can be computed by evaluating only one exponential. The response of the filter to all previous impulses is entirely accounted for with the weighting procedure just described. Consequently, the exponential must be evaluated only for those times at which time each impulse occurs, as well as at the time t of interest. A similar computational technique is described in Reference (2).

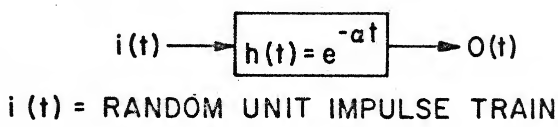

Figure 4 — Single order filter

Another important feature of this procedure provides for continuous generation of noise and does not require the use of fixed spacing integration procedures. This is a distinct advantage of the noise model since the sampling interval in many of the integration routines used in sophisticated digital simulations is controlled by computation accuracy and speed requirements.

Campbell's theorem

A noise $I(t)$ which is a superposition of pulses occurring at random times $\ldots t_{-1}, t_o, t_1, t_2, \ldots$ with all pulses of the same shape can be represented as

$$I(t) = \sum_i a_i h(t - t_i),$$

where $h(t)$ is the impulse response of the noise shaping filter and a_i is the amplitude of the random impulse. This is essentially the form of shot noise considered by Rice[3] and others.

In the references it is shown that if the random time t_i are governed by the Poisson density function

$$P(K, \tau) = \frac{(\bar{n}\,\tau)^k\, e^{-\bar{n}\,\tau}}{K}$$

**The $\pm$ sign comes from the fact that any given impulse is equally likely positive or negative.

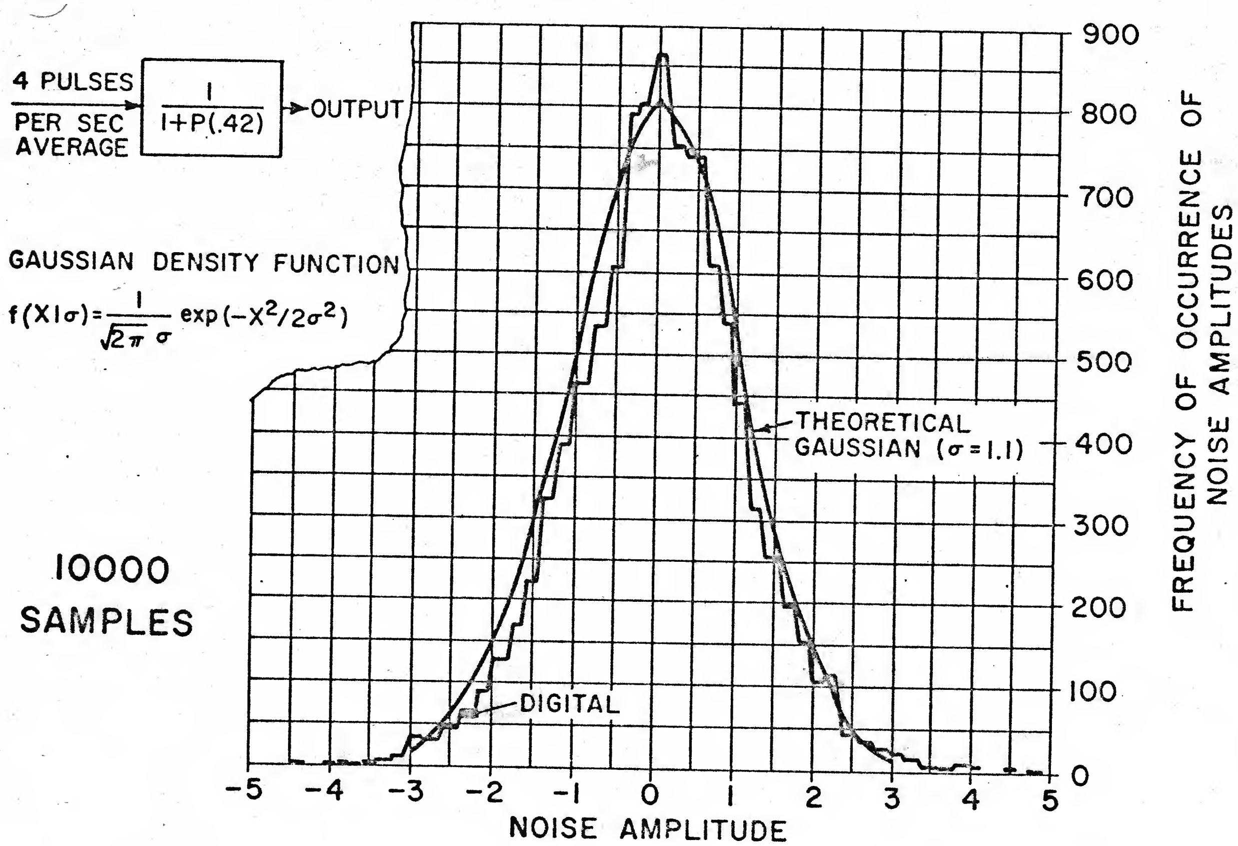

Figure 5 — Histogram of digital noise generators

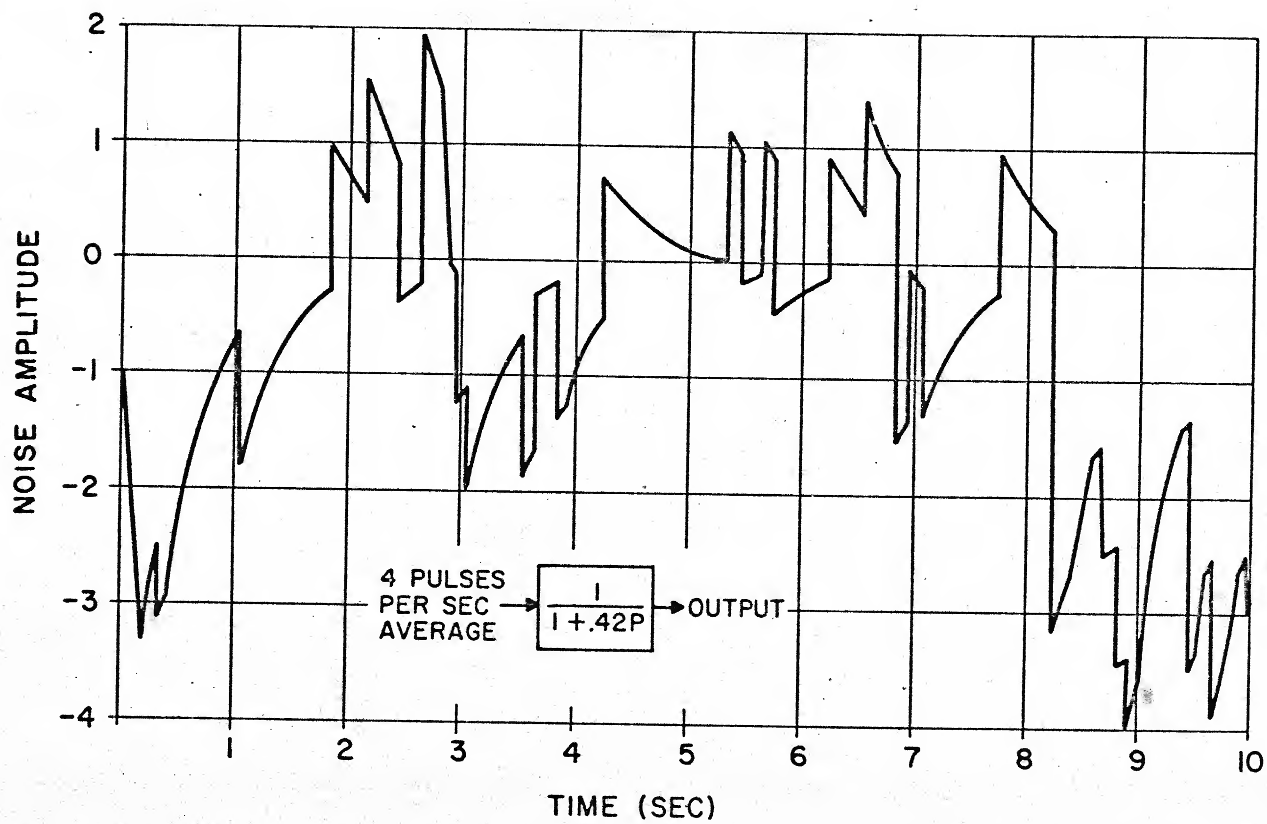

Figure 6 — Typical noise waveform

where $P(K, \tau)$ = probability of K events in the time interval τ

$\bar{n}$ = average number of events per unit time (an event is the occurrence of a pulse)

then the amplitude probability density function of I(t) becomes Gaussian as the ratio R becomes arbitrarily large where $R = \frac{\bar{n}}{\frac{\alpha}{2\pi}}$, and where $\frac{v}{1}$ = the time constant of low pass filter. (Experimentally we have shown that a ratio R of about 10 produces a noise signal with an amplitude probability density function which is very nearly Gaussian). Moreover it has been shown[3,5,6] that the autocorrelation of such a random wave is:

$$R_I(\tau) = \bar{n} \int_0^\infty c^2 e^{-\alpha t} e^{-\alpha(t+\tau)} dt = \frac{c^2 \bar{n} e^{-\alpha\tau}}{2\alpha} \quad (12)$$

with the corresponding spectral density

$$\Phi_I(\omega) = \frac{\bar{n} c^2}{\omega^2 + \alpha^2} \quad (13)$$

Performance of a typical noise generator

The technique described above was implemented as follows:

a) The times of occurrence $t_1, t_2, \ldots t_n, \ldots$ of a sequence of Poisson distributed impulses are generated

b) The output of the filter at any time t is computed as:

$$O(t) = \left[O(t_n) \pm c \right] e^{-\alpha(t-t_n)} \quad t_n \leqslant t < t_{n+1}$$

This output O(t) represents the output of a filter with impulse response $h(t) = ce^{-\alpha t}$ due to a sequence of Poisson distributed unit impulses which occur at times $t_1, t_2, \ldots t_n, \ldots$, each of which is equally likely positive or negative; $ce^{-\alpha(t-t_n)}$ denotes the contribution of the most recent (equally likely positive or negative) impulse to the output of the filter.

This scheme was coded in such a manner as to permit the simultaneous operation of 10 independent noise generators. A filter time constant of 0.42 sec, and an $\bar{n}$ of 4 pulses/sec were selected. This combination gives an R of about 10. The noise generators were sampled at 2 sec intervals for a total of 1000 samples each and the results combined to form the histogram of Figure 5. Note the close agreement between the experimental results and the theoretical Gaussian density function. The theoretical value of the standard deviation of the output of the filter has been determined to be 1.09 using the technique of Reference 6. Table I shows the mean and one sigma values of the noise generated by each of the 10 generators. Again, there is a close agreement with theoretical results.

Table I — Statistical performance of noise generators

Mean	Standard Deviation
−0.040	1.122
−0.0356	1.076
+0.0732	1.11
−0.00434	1.145
+0.01596	1.1
+0.0622	1.12
−0.0398	1.04
−0.001455	1.006
−0.05408	1.097
+0.00777	1.054

Theoretical mean = 0
Theoretical standard deviation = 1.1

A typical noise waveform is shown in Figure 6.

The autocorrelation function of the output of a noise generator is shown in Figure 7 along with the $\pm 3\sigma$ *** limits of the expected variation in the correlation function due to the sample size used[7]. Figure 8 shows the crosscorrelation function of two independent noise generators.

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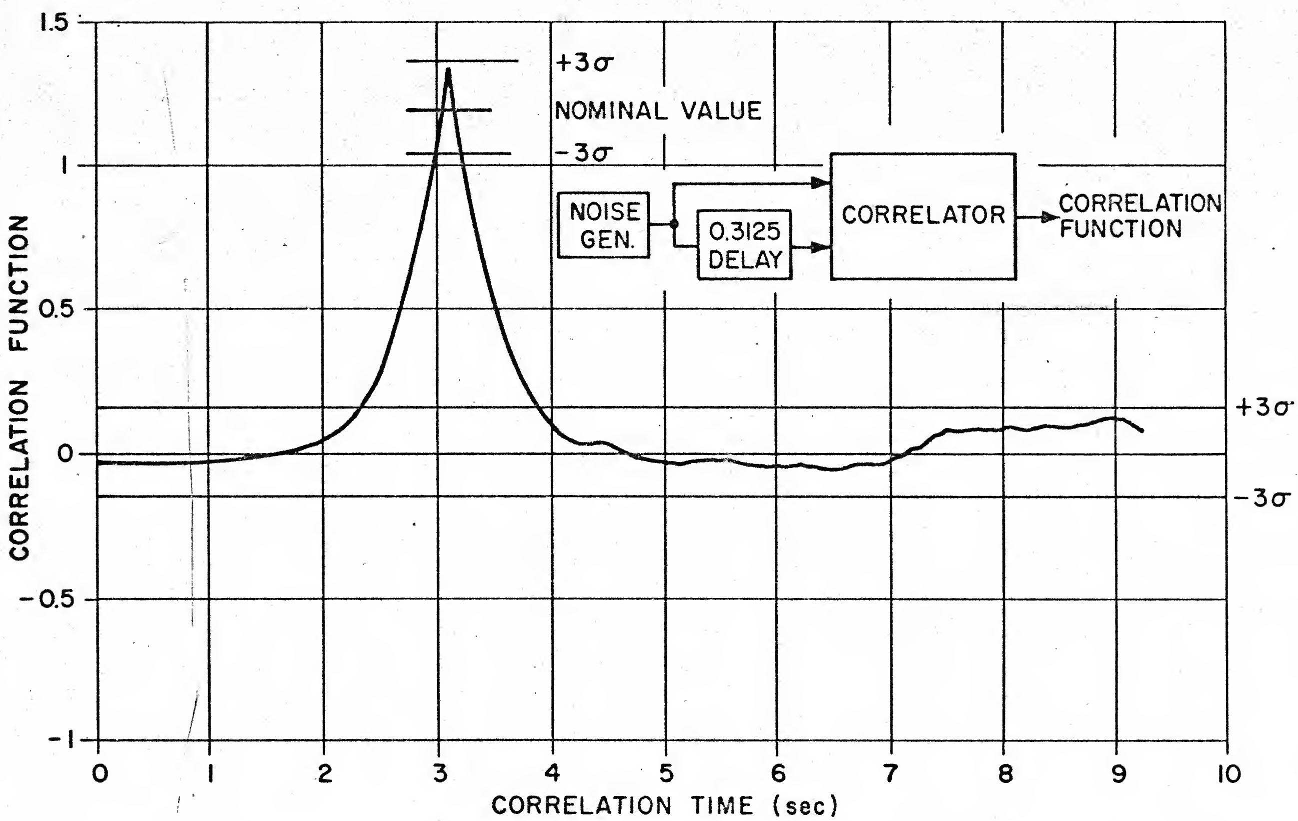

Figure 7 — Autocorrelation of noise generator

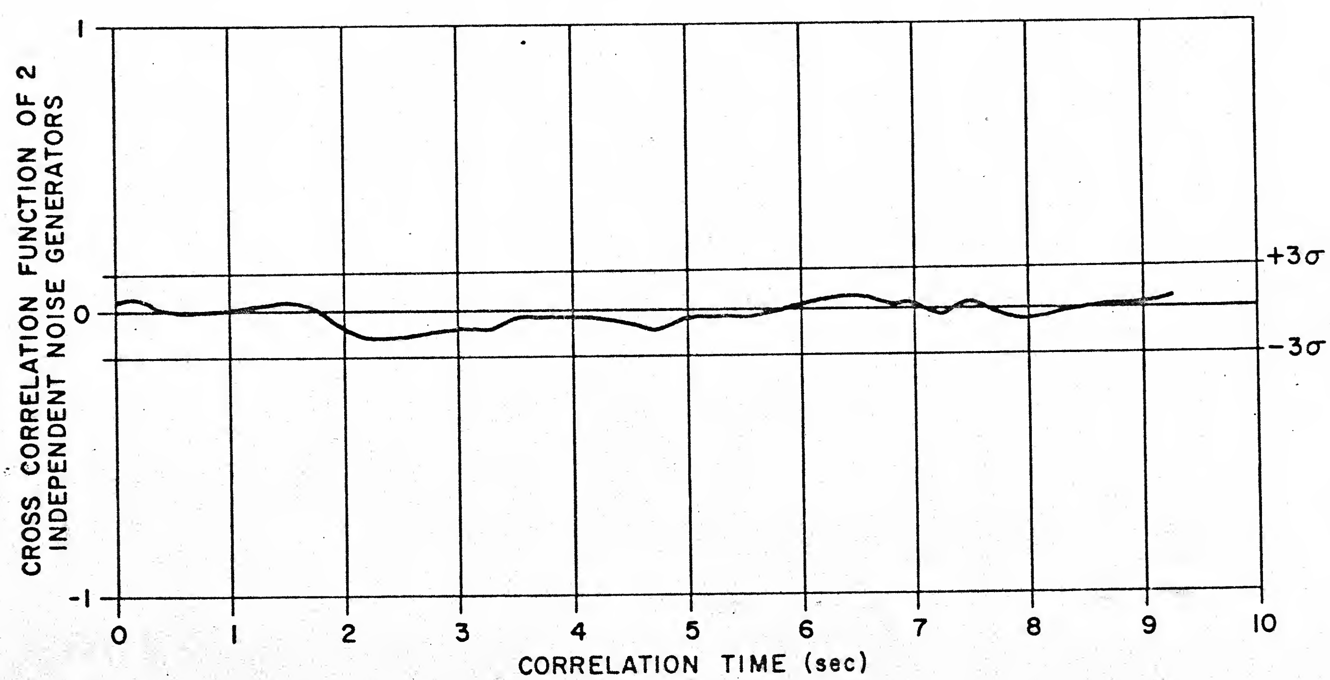

Figure 8 — Cross correlation between 2 noise generators